Question	Scheme	Marks	AOs
1(a)	$\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = ...,\ B = ...$	M1	1.1b
	Either $A = 2$ or $B = -1$	A1	1.1b
	$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{V}\mathrm{d}V = \int \frac{3}{(2t-1)(t+1)}\ \mathrm{d}t$	B1	1.1a
	$\int \frac{2}{2t-1} - \frac{1}{t+1}\mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\ln V = \ln(2t-1) - \ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$	M1	3.4
	$\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
	(b) Alternative separation of variables:		
	$\int \frac{1}{3V}\mathrm{d}V = \int \frac{1}{(2t-1)(t+1)}\ \mathrm{d}t$	B1	1.1a
	$\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1}\mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$	M1	3.1a
	$\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$	A1ft	1.1b
	Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$	M1	3.4
	$\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$ $V = \frac{3(2t-1)}{(t+1)}$ *	A1*	2.1
		(5)	
(c)	(i) 30 (minutes)	B1	3.2a
	(ii) 6 (m^3)	B1	3.4
		(2)	
		(10 marks)	
Notes:			

(a)

M1: Correct method of partial fractions leading to values for their A and B

E.g. substitution: $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = ..., B = ...$

Or compare coefficients $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A - B \Rightarrow A = ..., B = ...$

Note that $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = ..., B = ...$ scores M0

A1: Correct value for "A" or "B"

A1: Correct partial fractions not just values for "A" and "B". $\frac{2}{2x-1}-\frac{1}{x+1}$ or e.g. $\frac{2}{2x-1}+\frac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

B1: Separates variables $\int \frac{1}{V}\mathrm{d}V = \int \frac{3}{(2t-1)(t+1)}\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2t – 1) and/or the (t + 1) for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for +c here.

The brackets around the (2t – 1) and/or the (t + 1) must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

Alternative:

B1: Separates variables $\int \frac{1}{3V}\mathrm{d}V = \int \frac{1}{(2t-1)(t+1)}\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the dV and dt must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

M1: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2t – 1) and/or the (t + 1) for this mark

A1ft: Fully correct equation following through their A and B **only**.

No requirement for +c here.

The brackets around the (2t – 1) and/or the (t + 1) must be seen or implied for this mark

M1: Attempts to find "c" or e.g. "ln k" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

A1*: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3}\ln 3V=\frac{1}{3}\ln(2t-1)-\frac{1}{3}\ln(t+1)+c,\ \frac{1}{3}\ln 9=\frac{1}{3}\ln(3)-\frac{1}{3}\ln 3+c,\ c=\frac{1}{3}\ln 9$$

$$\ln 3V=\ln\frac{9(2t-1)}{(t+1)}\Rightarrow 3V=\frac{9(2t-1)}{(t+1)}\Rightarrow V=\frac{3(2t-1)}{(t+1)}*)$$

Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.

Note also that some candidates may use different variables in (b) e.g.

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3y}{(2x-1)(x+1)}\Rightarrow \int \frac{1}{y}\mathrm{d}y=\int \frac{3}{(2x-1)(x+1)}\mathrm{d}x$ etc. In such cases you should award marks for equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a "t" becomes an "x" within their working but is recovered allow full marks.

(c)

B1: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. ½ an hour.
If units are given they must be correct so do not allow e.g. 30 hours.

B1: Deduces 6 m^3. Units not required so just look for 6. Condone $V < 6$ or $V \leq 6$
If units are given they must be correct so do not allow e.g. 6 m.

Question	Scheme	Marks	AOs
2(a)	$\frac{\mathrm{d}V}{\mathrm{d}h}=200$ oe e.g. $\frac{\mathrm{d}h}{\mathrm{d}V}=\frac{1}{200}$	B1	1.1b
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow ...h^{\frac{3}{2}}=\lambda t\{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times 0+c\Rightarrow c=1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times 8+"1.152"\Rightarrow\lambda=0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
	(b) Alternative:		
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}h}=\frac{\sqrt{h}}{\lambda}\Rightarrow t=...h^{\frac{3}{2}}(+c)$	M1	1.1b
	$t=\frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	A1	1.1b
	$0=\frac{2(1.44)^{\frac{3}{2}}}{3\lambda}+c$ **and** $8=\frac{2(3.24)^{\frac{3}{2}}}{3\lambda}+c$ $\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **or** $c=...\left(-\frac{64}{19}\right)$	dM1	3.4
	$\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **and** $c=...\left(-\frac{64}{19}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
(c)	$5^{\frac{3}{2}}=0.513t+1.728\Rightarrow t=...$	M1	3.4
	$(t=)$ awrt18.4 min	A1	3.2a
		(2)	
		(10 marks)	

Notes

(a)

B1: For $\frac{dV}{dh} = 200$ stated or used – may be implied by their chain rule attempt

M1: Requires:

- $\frac{dV}{dh} = p, \quad p > 1$
- $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ (or a suitable letter for k, which may be λ, but must **not** be a number)
- **application** of the correct chain rule $\left(\frac{dh}{dt} =\right)\frac{dh}{dV} \times \frac{dV}{dt}$ or any equivalent with $\frac{dV}{dt} = \pm\frac{k}{\sqrt{h}}$ or $\pm\frac{1}{k\sqrt{h}}$ and their $\frac{dV}{dh}$ correctly placed. So $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{200k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores M0 as $\frac{dh}{dV}$ is incorrectly placed.

A1*: A rigorous argument with all steps shown and simplifies to achieve $\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$ with no errors.

Do not allow the use of λ for both constants. Allow use of e.g. $\frac{dV}{dt} = \pm\frac{1}{k\sqrt{h}}$ for full marks.

e.g. $\frac{dV}{dh} = 200, \quad \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{\lambda}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1M1A0* *unless* e.g. "let $\lambda = \frac{\lambda}{200}$ " seen.

Allow correct work leading to e.g. $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} \to \frac{\lambda}{\sqrt{h}}$ or $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ so $\lambda = \frac{k}{200}$

There must be an attempt to link the $\frac{dh}{dt}$ with the $\frac{\lambda}{\sqrt{h}}$ which may be missing an = sign.

Allow an argument with $\frac{dV}{dt} = -\frac{k}{\sqrt{h}}$ e.g. $\frac{dV}{dh} = 200, \quad \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times -\frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$

Withhold this A mark if there are notational errors e.g. $\frac{dV}{dt} = 200, \quad \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}}$ scores B1(implied)M1A0*

(b) Note that some candidates may work with e.g. $\lambda = \frac{k}{200}$ or e.g. $\lambda = 200k$ **which is acceptable.**

Candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ **then only the first 2 method marks are available (see note below). Condone use of other variables if the intention is clear but the final answer must be in terms of *h* and *t*.**

M1: Separates the variables and integrates to obtain an equation of the form $...h^{\frac{3}{2}} = \lambda t\{+c\}$ oe

The constant of integration is not needed for this mark.

A1: $\frac{2}{3}h^{\frac{3}{2}} = \lambda t(+c)$ oe. The constant of integration is not needed for this mark.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Substitutes $t = 0$ and $h = 1.44$ and attempts to find c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find "c" as long as they are using $t = 0$ and $h = 1.44$

May be implied by their value of c.

ddM1: Substitutes $t = 8$ and $h = 3.24$ and their c and attempts to find λ. Do not be concerned with the "processing" to find λ as long as they are using $t = 8$ and $h = 3.24$.

It is dependent on both previous method marks.

A1: Correct equation in the correct form from correct work. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Note candidates who do not have a λ e.g. assume $\frac{dh}{dt} = \frac{1}{200\sqrt{h}}$ or e.g. $\frac{dh}{dt} = \frac{200}{\sqrt{h}}$ can use either $t = 0$ and $h = 1.44$ **or** $t = 8$ and $h = 3.24$ to find their constant of integration.

(b)Alternative:

M1: Finds the reciprocal of both sides and integrates to obtain an equation of the form $t = ...h^{\frac{3}{2}}(+c)$

A1: $t = \dfrac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe. The constant of integration is not needed for this mark.

dM1: Substitutes $t=0$ and $h=1.44$ **and** substitutes $t=8$ and $h=3.24$ **and** attempts to find λ **or** c.
It is dependent on the previous method mark.
Do not be concerned with the "processing" to find λ or c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$ and reach a value for λ or c. May be implied by their value(s).

ddM1: Complete attempt to find λ **and** c. **It is dependent on both previous method marks.**
Do not be concerned with the "processing" to find λ and c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$.

A1: Correct equation in the correct form. $h^{\frac{3}{2}} = 0.513t + 1.728$ or $h^{\frac{3}{2}} = \dfrac{513}{1000}t + \dfrac{216}{125}$
Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Special Case:

Some candidates are using the given equation in part (b) to find the value of A and the value of B using the given conditions. May score a maximum of 00110. This should be marked as follows:

M0A0: (No attempt to integrate)
M1: Substitutes $t = 0$ and $h = 1.44$ to find a value for B
dM1: Substitutes $t = 8$ and $h = 3.24$ with their value of B to find a value for A
A0: Since they have not used the given model.
(Allow full recovery in (c) if this equation is correct)

(c)

M1: Attempts to substitute $h = 5$ into their equation which must be of the form $h^{\frac{3}{2}} = At + B$ or possibly a rearranged equation e.g. $h^{\frac{1}{2}} = \sqrt[3]{At + B}$ with values of A and B leading to a value for t.
Do not be concerned about the processing as long as they use $h = 5$ and obtain a value for t even if t is negative.

A1: Awrt 18.4 minutes **following a correct equation in (b)**.
The units are required but allow e.g. min, mins, but not just 18.4 and not m (which means metres)
Allow e.g. 18 minutes 25 seconds or 18 mins 26 secs
Note this may follow A0 in part (b) as they may have rearranged incorrectly in (b) but use a correct equation in (c) e.g. $\dfrac{2}{3}h^{\frac{3}{2}} = \dfrac{171}{500}t + \dfrac{144}{125}$, $h^{\frac{1}{2}} = \sqrt[3]{\dfrac{513}{1000}t + \dfrac{216}{125}}$ or may come from the special case.
Apply isw following a correct time and units, e.g., 18.4 followed by 18 mins.